ONE-DIMENSIONAL CENTRAL-FORCE PROBLEM FOR SOMMERFELD SPHERE IN CLASSICAL ELECTRODYNAMICS: SOME NUMERICAL RESULTS

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Equation of motion of Sommerfeld sphere in the field of Coulomb center is numerically investigated. It is shown that contrary to Lorentz-Dirac equation in the attractive case there are physical solutions. In the repulsive case sphere gains less energy then that should be according to relativistic equation of motion of point charge without radiation force.

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Numerical calculations of head-on collisions of two point charged particles in classical electrodynamics with retardation and radiation reaction show many interesting properties of Lorentz-Dirac equation [1-6].

Among them are:

(P.1) absence of physical trajectory in the attractive case - for finite initial values of position, energy and acceleration point charge stops before it reaches the Coulomb center of the opposite sign and then turns back and moves away to infinity with velocity growing up to that of light [1,4,6];

(P.2) in the repulsive case point charge can gain velocity, after the turning point, much more greater then that follows from the relativistic equations of motion without radiation force [4].

These and other effects (among them is the effect of preacceleration) cause much doubt in validity of standard approach to radiation reaction.

In literature one can find the opinion that only consistent quantum theory can solve all problems of radiation reaction [7];

but also one can find the point of view that the problems lie in the first principles of classical theory, for example, in the notion of "point" particle (quantum theory only rewrites classical problems in another language), and for "extended" (in some sense) particles the situation will be different [8-11].

From the latter point of view it is interesting to consider how the above results of numerical calculations change for "extended", not "point-like" charges. For this sake lets consider the famous Sommerfeld model of extended charge with self-action.

Long time ago in Sommerfeld works [12, see also 9,13] was derived the expres-

sion of self-force acting on "nonrelativistically rigid charged sphere", i.e sphere with radius a, its center moving along trajectory $\vec{R}(t)$, with total charge Q and charge density (in laboratory reference frame)

$$\rho(t, \vec{r}) = \frac{Q}{4\pi a^2} \delta(|\vec{r} - \vec{R}| - a).$$

(One can treat this model in the following way: one builds the uniformly charged sphere in laboratory reference frame and then begins it to accelerate in the way that the charge density in laboratory frame is described by the above equation while in sphere self-frame charge density can be calculated by standard tensor coordinate transformations.)

In the case of shell rectilinear motion this force has the form [9,13]

$$F_{self} = \frac{Q^2}{4a^2} \left[-c \int\limits_{T^-}^{T^+} dT \frac{cT - 2a}{L^2} + \ln \frac{L^+}{L^-} + (\frac{1}{\beta^2} - 1) \ln \frac{1+\beta}{1-\beta} - \frac{2}{\beta} \right] \tag{1}$$

here $cT^\pm = 2a \pm L^\pm, \quad L^\pm = R(t) - R(t - T^\pm), \quad L = R(t) - R(t - T), \quad \beta = v/c, \quad v = dR/dt.$

The total shell equation of motion then will be

$$m \frac{d}{dt}(\gamma v) = F_{self} \tag{2}$$

Here m - is the "mechanical" shell mass.

This equation has one trivial solution - the uniform motion without radiation: $R(t) = R_0 + vt$.

Introducing dimensionless variables $y = R/2a, \quad x = ct/2a$ one can rewrite the shell equation of motion (2) in the form

$$\frac{d^2 y}{dx^2} = \left(1 - (\frac{dy}{dx})^2 \right)^{3/2} k \cdot$$

$$\cdot \left[- \int\limits_{x^-}^{x^+} dz \frac{z - 1}{L^2} + \ln \frac{L^+}{L^-} + (\frac{1}{\beta^2} - 1) \ln \frac{1+\beta}{1-\beta} - \frac{2}{\beta} \right] \tag{3}$$

here

$$x^\pm = 1 \pm L^\pm, \quad L^\pm = y(x) - y(x - x^\pm), \quad L = y(x) - y(x - z),$$

$$\beta = dy/dx, \qquad k = \frac{Q^2}{2mc^2 a}.$$

Lets take the charged sphere of diameter $2a$ equal to the "particle radius" $\frac{Q^2}{mc^2}$:

$$k = 1.$$

Lets place this sphere into the Coulomb field of charge q. Then the equation of motion of such central-force problem reads

$$\frac{d^2y}{dx^2} = \left(1 - (\frac{dy}{dx})^2\right)^{3/2} \cdot$$

$$\cdot \left[-\int_{x^-}^{x^+} dz \frac{z-1}{L^2} + \ln\frac{L^+}{L^-} + (\frac{1}{\beta^2} - 1)\ln\frac{1+\beta}{1-\beta} - \frac{2}{\beta} + \frac{M}{(y-d)^2} \right] \tag{4}$$

here d - is coordinate of Coulomb center, $M = q/Q$.

It is useful to compare solutions of (4) with point charge motion in the same field, governed by the following relativistic equation without radiation force:

$$\frac{d^2y}{dx^2} = \left(1 - (\frac{dy}{dx})^2\right)^{3/2} \cdot \left[\frac{M}{(y-d)^2}\right] \tag{5}$$

A.

We integrated eq.(4,5) in the repulsive case numerically with the following initial data:

(i) Coulomb center is placed at $d = 5.0$;

(ii) initial value of coordinates of the point particle and of sphere center of mass is $y = 0.0$;

(iii) initial sphere and point particle velocities $\frac{dy}{dx}$ are zero (and $\frac{dy}{dx} = 0.0$ for $x < 0.0$);

(iv) M is taken equal to 1.0 and to 0.1.

Numerical results are shown on figs. (A.1-A.3):

curves vz, vq correspond to velocities of Sommerfeld sphere and of point particle (Fig. A.1 for $M = 1.0$ and Fig. A.2 for $M = 0.1$);

curves wz, wq correspond to accelerations of Sommerfeld sphere and of point charge (Fig. A.3 for $M = 1.0$);

horizontal axis is x.

One can see that there is the following main property of motion of Sommerfeld sphere:

sphere gains velocity less then that should be according to relativistic equation of motion of point charge without radiation reaction.

This result one can explain as simple consequence of effect of retardation.

B.

In the attractive case we numerically intergated eq.(4,5) with the following initial data:

(i) Coulomb center is placed at $d = 5.0$;

(ii) initial value of coordinates of the point particle and of sphere center of mass is $y = 0.0$;

(iii) initial sphere and point particle velocities $\frac{dy}{dx}$ are zero (and $\frac{dy}{dx} = 0.0$ for $x < 0.0$);

(iv) M is taken equal to -1.0;

curve vz corresponds to velocity of Sommerfeld sphere;

curve vq corresponds to velocity of point charge;

horizontal axis is x.

Numerical results are shown on fig. (B.1).

One can see that Sommerfeld sphere indeed falls on the Coulomb center, so there is physical trajectory contrary to the motion of point charge governed by Lorentz-Dirac equation.

Thus we conclude that extended radiating object can solve problems of Lorentz-Dirac approach. This happens thanks to the fact that equations of motion of extended objects are not analytic near the zero value of their size ($a = 0$) and thus equations with $a = 0$ and $a \to 0$ are essentially different equations with different physical solutions.

I am glad to thank my colleagues:

P.A.Polyakov - for theoretical discussions;

V.A.Iljina and P.K.Silaev - for assistance rendered during numerical calculations.

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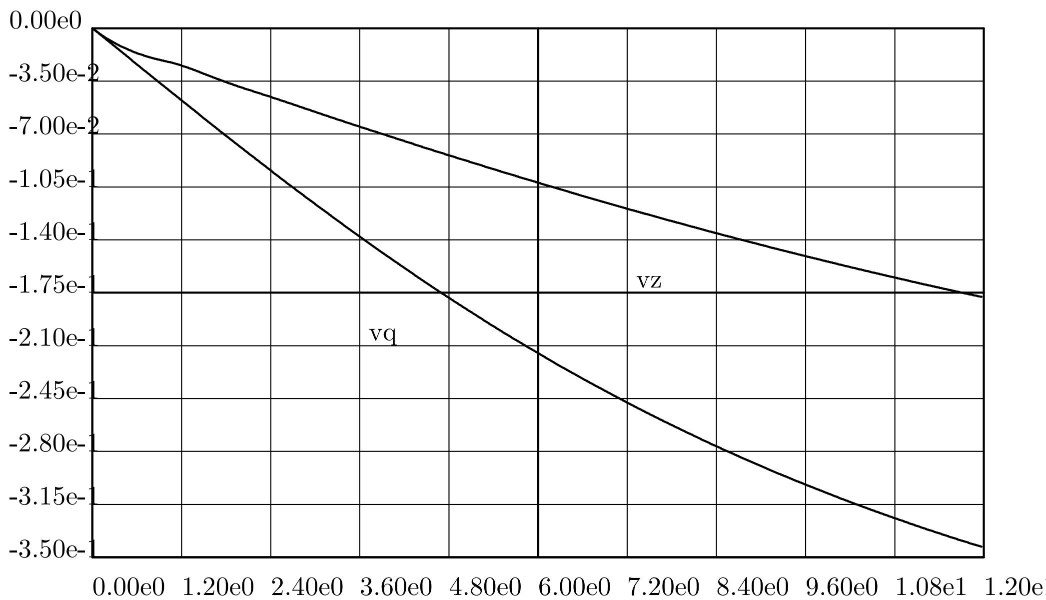
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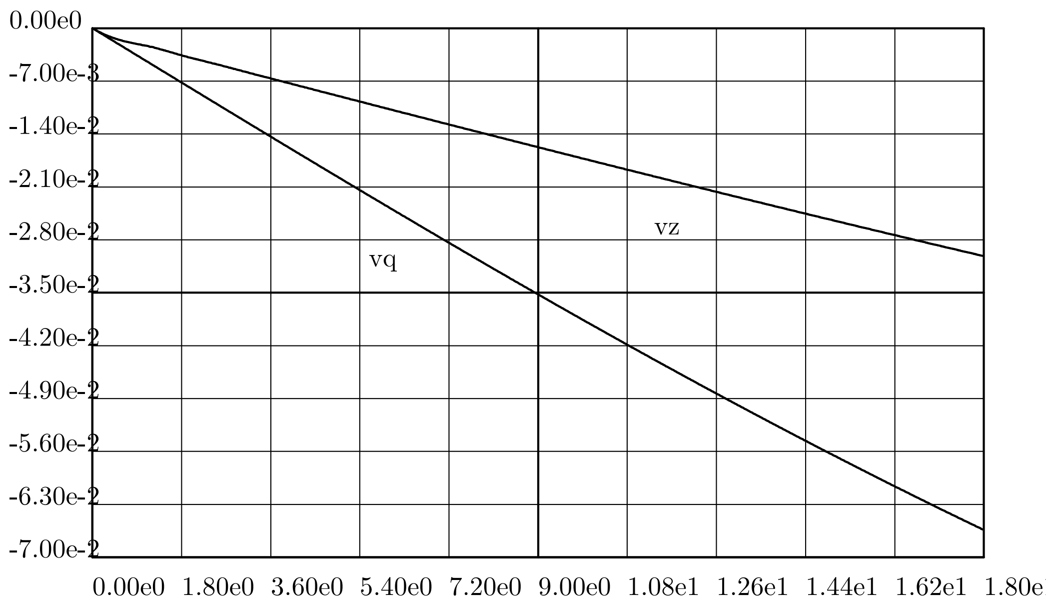
Fig. A. 1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vq
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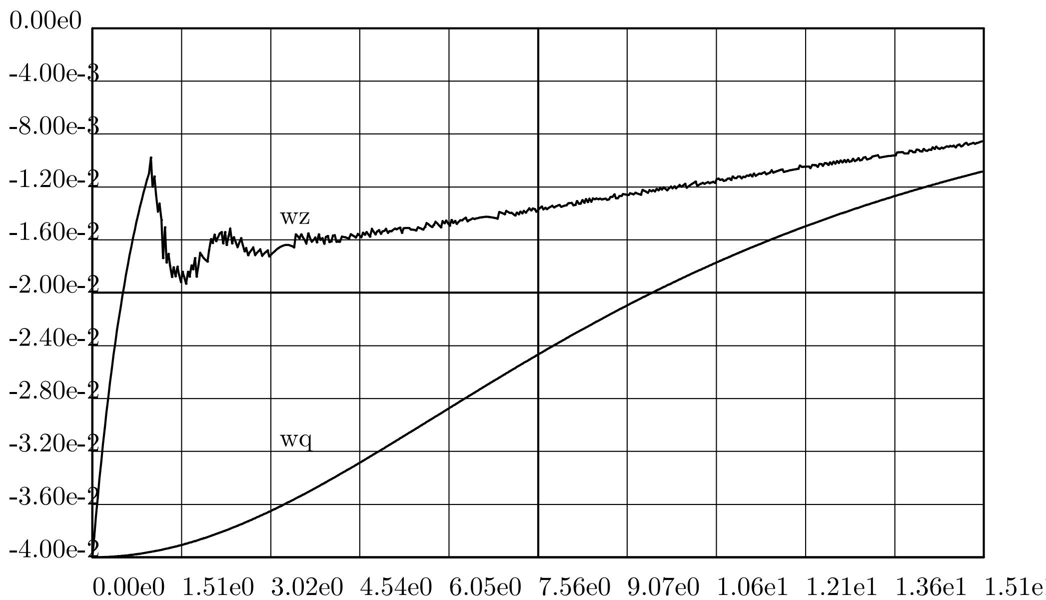
Fig. A. 2



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wz
wq
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3.02e0
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1.51e1

Fig. A. 3

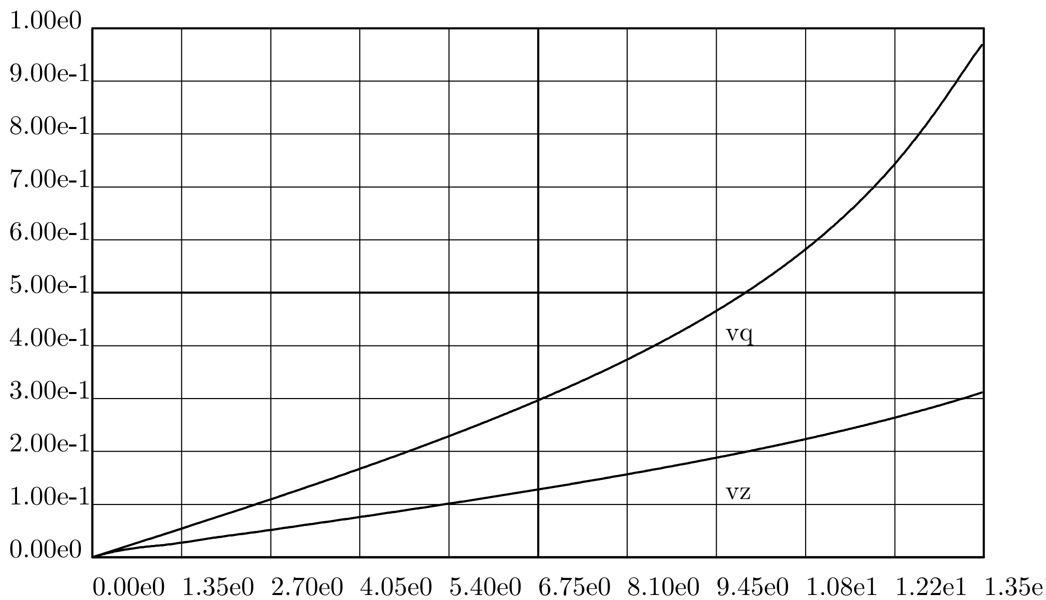
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Fig. B. 1